(2PG128)

S.C.No.—21703205

M. Sc. EXAMINATION, 2022

(Second Semester)

(Batch 2021)

MATHEMATICS

21MTH-205

Computational Techniques

Time 3 Hours   Maximum Marks 80

Note Attempt *Five* questions in all. All questions carry equal marks.

1. (a) Define Rounding error. Find the absolute error if the number X = 0.00545828 is rounded off to three decimal digits.

   (b) Find $\sqrt{5}$, correct to four decimal places by Newton's method.

(c) Evaluate $\Delta^3 f(2)$ from the data by constructing the table of differences

x	0	1	2	3	4
$f(x)$	1.0	1.5	2.2	3.1	4.6

(d) Given (x_1, y_1), (x_2, y_2), (x_3, y_3), write Lagrange's interpolation formula

(e) Use forward differences, the formula for $\dfrac{dy}{dx}$

(f) $f(x)$ is given by

x	0	0.5	1	1.5	2.0
$f(x)$	1	0.8	0.5	0.25	0.10

Evaluate $\int\limits_{0}^{2} f(x)\,dx$, using Trapezoidal rule.

(g) Solve $\dfrac{dy}{dx} = x + y$, $y(1) = 0$ at 1.1, using Taylor's series method.

(b) Solve the difference equation :

$$u_{n+3} - 2u_{n+2} - 5u_{n+1} + 6u_n = 0.$$

2. (a) Find the absolute and relative errors if the number 37.46235 is (i) rounded off to four significant figures and (ii) truncated to three decimal digits.

(b) Solve, by Jacobi's iteration method, the equations :

$$20x + y - 2z = 17 ;$$
$$3x + 20y - z = -18 ;$$
$$2x - 3y + 20z = 25 .$$

3. (a) Find a root of the equation $x^3 - 2x - 5 = 0$, using secant method, correct to three decimal places.

(b) Discuss convergence of Newton-Raphson method and find a root of equation $x^3 - 3x + 1 = 0$ by this method, correct to two decimal places.

4. (a) If $y_{10} = 3$, $y_{11} = 6$, $y_{12} = 11$, $y_{13} = 18$, $y_{14} = 27$, find y_4.

(b) Find the cubic polynomial which takes the following values :

x	0	1	2	3
$f(x)$	1	2	1	10

Hence or otherwise evaluate $f(4)$.

5. (a) Find the polynomial $f(x)$ by using Lagrange's formula :

x	0	1	3	6
$f(x)$	18	10	-18	90

Also find the slope of the curve at $x = 2$

(b) The following values of x and y are given

x	1	2	3	4
y	1	2	5	11

Find the cubic splines and evaluate $y(1.5)$.

6. (a) Find $y'(0)$ and $y''(0)$ from the following table :

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(b) Using the Richardson extrapolation method, find $y''(0.6)$ from the following tabulated function by applying the formula

$$F(x) = \frac{1}{h^2}\left[y(x+h) - 2y(x) + y(x-h) \right]$$

with $h = 0.4, 0.2, 0.1$

x	0.2	0.4	0.5	0.6	0.7	0.8	1.0
y(x)	1.42	1.88	2.13	2.39	2.66	2.94	3.56

(a) Evaluate $\int_0^{12} \frac{dx}{1+x^2}$ by using Trapezoidal rule with $h = 2$ and by Simpson's $\frac{1}{3}$rd rule with $h = 2$

(b) Use Romberg's method to compute

$$\int_0^1 \frac{dx}{1+x}$$ correct to three decimal places.

8. (a) Solve the differential equation

$$\frac{dy}{dx} = -xy^2, \quad y = 2 \text{ at } x = 0$$

by modified Euler's method and obtain y at $x = 0.2$ in two steps of 0.1 each

(b) Given $\frac{dy}{dx} = 1 + y^2$; $y(0) = 0$, find $y(0.2)$ and $y(0.4)$, taking $h = 0.2$

9. (a) Find the largest eigen-value and the corresponding eigen-vector, by Power method of the matrix $\begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$

(b) Solve the difference equation :

$$u_{n+2} - 2\cos\alpha\, u_{n+1} + u_n = \cos\alpha n.$$