(MR319) Roll No. ....

S.C.No.—A/21/2009203

B. Sc. (Honours) EXAMINATION, 2021

(Second Semester)

VECTOR CALCULUS

BHM123

Time : 2 Hours *Maximum Marks : 60*

Note : Attempt *Four* questions in all. All questions carry equal marks.

Section I

1. (a) Find the value of λ, so that the following vectors are coplanar :

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k},$$

$$\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k},$$

$$\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}.$$ 7.5

(b) If $\vec{a}$, $\vec{b}$, $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$; find the angles which $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$, where $\vec{b}$ and $\vec{c}$ are non-parallel. 7.5

2. (a) The necessary and sufficient condition for the vector function $\vec{f}$ of a scalar variable t to have constant direction is $\vec{f} \times \frac{d\vec{f}}{dt} = 0$. 7.5

(b) A particle moves along the curve $x = 3t^2$, $y = t^2 - 2t$, $z = t^3$. Find its velocity and acceleration at $t = 1$ in the direction of vector $\vec{a} = \hat{i} + \hat{j} - \hat{k}$. 7.5

Section II

3. (a) If $\vec{a} = x^2yz\,\hat{i} - 2xz^3\,\hat{j} + xz^2\,\hat{k}$ and $\vec{b} = 2z\,\hat{i} + y\,\hat{j} - x^2\,\hat{k}$, find $\frac{\partial^2}{\partial x \partial y}(\vec{a} \times \vec{b})$ at the point $(1, 0, -2)$. 7.5

(b) If $r = |\vec{r}|$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that :

$$\vec{a} \cdot \nabla\left(\vec{b} \cdot \nabla \frac{1}{r}\right) = \frac{3(\vec{a} \cdot \vec{r})(\vec{b} \cdot \vec{r})}{r^5} - \frac{\vec{a} \cdot \vec{b}}{r^3}.$$

7.5

4. (a) Find the directional derivative of :

$$\phi(x, y, z) = x^2yz + 4xz^2$$

at the point (1, – 2, 1) in the direction $2\hat{i} - \hat{j} - 2\hat{k}$. 7.5

(b) Show that :

$$\text{div}\left[\frac{f(r)\vec{r}}{r}\right] = \frac{1}{r^2} \cdot \frac{d}{dr}\left[r^2 f(r)\right].$$ 7.5

Section III

5. (a) Determine the transformation from cylindrical to rectangular coordinates. 7.5

(b) Express $x\hat{i} + 2y\hat{j} + yz\hat{k}$ in spherical coordinates. 7.5

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6. (a) If (r, θ, ϕ) are spherical co-ordinates, show that $\nabla\left(\frac{1}{r}\right) = \nabla \times (\cos\theta \nabla\phi)$. 7.5

(b) Express the velocity $\vec{v}$ and acceleration $\vec{a}$ of a particle in cylindrical co-ordinates. 7.5

Section IV

7. (a) If $\vec{r} = 2t\hat{i} + 3t^2\hat{j} - t^3\hat{k}$; evaluate :

$$\int_1^2 \left(\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}\right) dt.$$ 7.5

(b) Evaluate $\iint_S \phi \hat{n}\, ds$, where $\phi = \frac{3}{8}xyz$ and S is the surface of cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ to $z = 5$. 7.5

8. (a) If S is any closed surface enclosing a volume V and $\vec{f} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$, then show that $\iint_S \vec{f} \cdot \hat{n}\, ds = 6V$. 7.5

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(b) State and prove Stokes' theorem. **7.5**

Section V

9. (a) Define reciprocal system of vectors.

(b) Find a unit tangent vector to any point on the curve $x = a\cos t$, $y = a\sin t$, $z = bt$.

(c) Find 'a' so that the vectors :

$$\vec{f} = \left(axy - z^3\right)\hat{i} + (a-2)x^2\,\hat{j} + (1-a)x^2 z\,\hat{k}$$

is irrotational.

(d) Show that $\iint_S \hat{n}\, ds = 0$ for any closed surfaces.

(e) Define cylindrical co-ordinates.

(f) Show that $\oint_C \vec{r} \cdot d\vec{r} = 0$. **2.5×6=15**