(UG760) **Roll No. ....**

S.C.No.—2009102

B. Sc. (Hons.) EXAMINATION, 2021

(First Semester)

CALCULUS

BHM-112

Time : 2 Hours *Maximum Marks : 60*

Note : Attempt any *Four* questions. All questions carry equal marks.

1. (a) Expand $\log(1+x)$ by Maclaurin's series.

$\log(1+x)$

(b) If $y = \sin\left(m \sin^{-1} x\right)$, prove that :

$$\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} + \left(m^2-n^2\right)y_n = 0$$

$y = \sin\left(m \sin^{-1} x\right)$

$$\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} + \left(m^2-n^2\right)y_n = 0$$

2. (a) State and prove Leibnitz theorem for the nth derivative of the product of two functions.

n

(b) By using Taylor's series prove that :

$$\sin(x+y) = \sin x \cos y + \cos x \sin y.$$

Hence find the value of $\sin 46^\circ$ correct to four places of decimals.

$$\sin(x+y) = \sin x \cos y + \cos x \sin y.$$

$\sin 46^\circ$

3. (a) Find the asymptotes of the curve : $(x+y)^2(x+y+2)=x+9y-2$.

$$(x+y)^2(x+y+2)=x+9y-2$$

(b) If ρ_1 and ρ_2 be the radii of curvature at the extremities of a focal chord of a parabola whose Latus rectum is '4a' then prove that :

$$\rho_1^{-2/3}+\rho_2^{-2/3}=2a^{-2/3}$$

ρ_1 ρ_2

'4a'

$$\rho_1^{-2/3}+\rho_2^{-2/3}=2a^{-2/3}$$

4. (a) Find the position and nature of the double points on the curve $(y-x)^2+x^7=0$.

$$(y-x)^2+x^7=0$$

(b) Find the radius of curvature at any point of the curve $r^2=a^2\cos 2\theta$.

$$r^2=a^2\cos 2\theta$$

5. (a) Find the points of inflexion for the curve $54y=(x+5)^2(x^3-10)$.

$$54y=(x+5)^2(x^3-10)$$

(b) Apply Newton's formula to find the radius of curvature at the origin of the cycloid $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$.

$$x=a(\theta+\sin\theta), y=a(1-\cos\theta)$$

6. (a) Trace the curve $x=a(\theta-\sin\theta)$, $y=(1-\cos\theta)$; $0\le\theta\le 2\pi$.

$x = a(\theta - \sin\theta)$, $y=(1-\cos\theta)$;

$0\le\theta\le 2\pi$.

(b) Find the reduction formula for $\int \sec^n x dx$.

$\int \sec^n x dx$

7. (a) Find the length of the arc $x^2 + y^2 - 2ax = 0$ in the first quadrent.

$$x^2 + y^2 - 2ax = 0$$

(b) Evaluate :

$$\int_0^{2\pi} x^3 \left(\sqrt{2ax - x^2}\right) dx$$

$$\int_0^{2\pi} x^3 \left(\sqrt{2ax - x^2}\right) dx$$

8. (a) Find the volume of the solid generated by revolving the ellipse about the major axis.

(b) Find the area lying inside the circle $r = a\sin\theta$ and outside the cardiod $r = a(1 - \cos\theta)$.

$r = a\sin\theta$

$r = a(1 - \cos\theta)$

9. (a) Find the centroid of the semicircular region of radius r by Pappus theorem.

r

(b) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$.

$y^2 = 4ax$ $x^2 = 4ay$

$\frac{16}{3}a^2$